Surface Areas and Volumes

Surface Areas of Cubes and Cuboids

Surface Areas of a Cube and a Cuboid

We give gifts to our friends and relatives at one time or another. We usually wrap our gifts in nice and colourful wrapping papers. Look, for example, at the nicely wrapped and tied gift shown below.

Clearly, the gift is packed in box that is cubical or shaped like a **cube**. Suppose you have a gift packed in a similar box. How would you determine the amount of wrapping paper needed to wrap the gift? You could do so by making an estimate of the surface area of the box. In this case, the total area of all the faces of the box will tell us the area of the wrapping paper needed to cover the box.

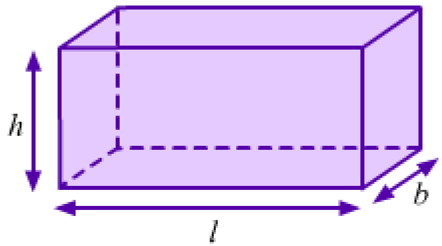
Knowledge of surface areas of the different solid figures proves useful in many real-life situations where we have to deal with them. In this lesson, we will learn the formulae for the surface areas of a cube and a **cuboid**. We will also solve some examples using these formulae.

Did You Know?

- The word 'cuboid' is made up of 'cube' and '-oid' (which means 'similar to'). So, a cuboid indicates something that is similar to a cube.
- A cuboid is also called a 'rectangular prism' or a 'rectangular parallelepiped'.

Formulae for the Surface Area of a Cuboid

Consider a cuboid of length l, breadth b and height h.



The formulae for the surface area of this cuboid are given as follows:

Lateral surface area of the cuboid = $2h(l + b)$

Total surface area of the cuboid = $2(lb + bh + hl)$

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

Did You Know?

Two mathematicians named Henri Lebesgue and Hermann Minkowski sought the definition of surface area at around the twentieth century.

Know Your Scientist

Henri Lebesgue (1875–1941) was a French mathematician who is famous for his theory of integration. His contribution is one of the major achievements of modern analysis which greatly expands the scope of Fourier analysis. He also made important contributions to topology, the potential theory, the *Dirichlet* problem, the calculus of variations, the set theory, the theory of surface area and the dimension theory.

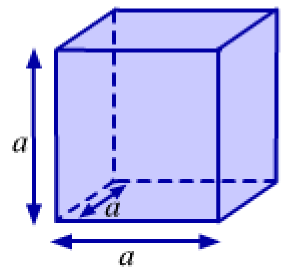
Hermann Minkowski (1864–1909) was a Polish mathematician who developed the geometry of numbers and made important contributions to the number theory, mathematical physics and the theory of relativity. His idea of combining time with the three dimensions of space, laid the mathematical foundations for Albert Einstein's theory of relativity.

Did You Know?

The concept of surface area is widely used in chemical kinetics, regulation of digestion, regulation of body temperature, etc.

Formulae for the Surface Area of a Cube

Consider a cube with edge *a*.



The formulae for the surface area of this cube are given as follows:

Lateral surface area of the cube = $4a^2$

Total surface area of the cube = $6a^2$

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

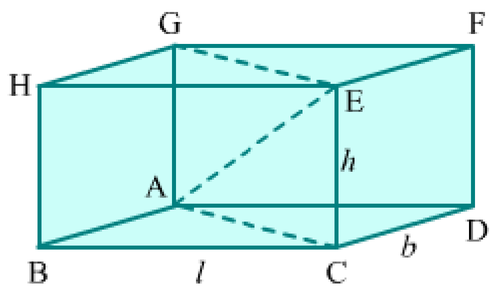
Did You Know?

- A cube can have 11 different nets.
- Cubes and cuboids are **convex polygons** that satisfy **Euler's formula**, i.e., **F + V − E = 2**.

Know More

Length of the diagonal in a cube and in a cuboid

A cuboid has four diagonals (say AE, BF, CG and DH). The four diagonals are equal in length.



Let us consider the diagonal AE.

In rectangle ABCD, length of diagonal AC = $\sqrt{l^2 + b^2}$

Now, ACEG is a rectangle with length AC and breadth CE or *h*.

So, length of diagonal AE = $\sqrt{AC^2 + CE^2}$

$$= \sqrt{\left(\sqrt{l^2 + b^2}\right)^2 + h^2}$$
$$= \sqrt{l^2 + b^2 + h^2}$$

∴ **Length of the diagonal of a cuboid =** $\sqrt{\boldsymbol{l^2 + b^2 + h^2}}$

A cube is a particular case of cuboid in which the length, breadth and height are equal to *a*.

∴ **Length of the diagonal of a cube** = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{\mathbf{3}}\boldsymbol{a}$

Solved Examples

Easy

Example 1: **There are twenty-five cuboid-shaped pillars in a building, each of dimensions 1 m ×1 m × 10 m. Find the cost of plastering the surface of all the pillars at the rate of Rs 16 per m².**

Solution:

Length (l) of one pillar = 1 m

Breadth (b) of one pillar = 1 m

Height (h) of one pillar = 10 m

∴ Lateral surface area of one pillar= $2h(l + b)$

= $2 \times 10 \times (1 + 1)$ m^2

= 40 m^2

⇒ Lateral surface area of twenty-five pillars = (25×40) m^2 = 1000 m^2

Cost of plastering 1 m^2 of surface = Rs 16

⇒ Cost of plastering 1000 m^2 of surface = Rs (16 × 1000) = Rs 16000

Thus, the cost of plastering the twenty-five pillars of the building is Rs 16000.

Example 2: **Find the length of the diagonal of a cube whose surface area is 294 m^2.**

Solution:

Let the edge of the given cube be a.

∴ Surface area of the cube = $6a^2$

It is given that the surface area of the cube is 294 m^2.

So, $6a^2 = 294$

$$\Rightarrow a^2 = 49 \text{ m}^2$$
$$\Rightarrow \therefore a = \sqrt{49} \text{ m} = 7 \text{ m}$$

Now, length of the diagonal of the cube = $\sqrt{3}a = 7\sqrt{3}$ m

Medium

Example 1: A metallic container (open at the top) is a cuboid of dimensions 7 cm × 5 cm × 8 cm. What amount of metal sheet went into making the container? Also, find the cost required for painting the outside of the container, excluding the base, at the rate of Rs 17 per 3 cm^2.

Solution:

Length (l) of the container = 7 cm

Breadth (b) of the container = 5 cm

Height (h) of the container = 8 cm

The container is open at the top. Therefore, while calculating the amount of metal sheet used, we will exclude the top part.

∴ Amount of metal sheet used = Total surface area − Area of the top part

$= 2\ (lb + bh + lh) - lb$

$= [2 \times (7 \times 5 + 5 \times 8 + 7 \times 8) - 7 \times 5]\ \text{cm}^2$

$= [2 \times (35 + 40 + 56) - 35]\ \text{cm}^2$

$= (2 \times 131 - 35)\ \text{cm}^2$

$= 227\ \text{cm}^2$

Thus, 227 cm^2 of metal went into making the given container.

Now, area to be painted = Lateral surface area of the cuboid

$= 2h\ (l + b)$

$= [2 \times 8 \times (7 + 5)]\ \text{cm}^2$

$= (16 \times 12)\ \text{cm}^2$

$= 192\ \text{cm}^2$

Cost of painting 3 cm^2 of surface = Rs 17

⇒ Cost of painting 1 cm^2 of surface = Rs $\frac{17}{3}$

⇒ Cost of painting 192 cm^2 of surface = Rs $\left(192 \times \frac{17}{3}\right)$ = Rs 1088

Therefore, the cost of painting the outside of the container is Rs 1088.

Example 2: If the total surface area of a cube is $24x^2$, then find the surface area of the cuboid formed by joining

i)two such cubes.

ii)three such cubes.

Solution:

Total surface area of cube = $6a^2$

It is given that the total surface area of the cube is $24x^2$.

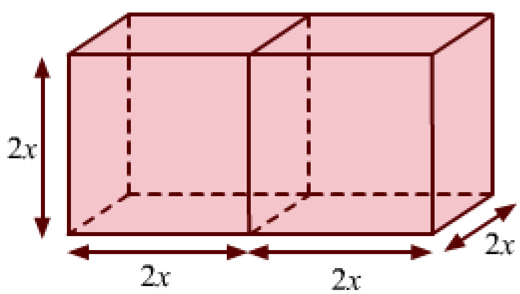
So, $6a^2 = 24x^2$

$\Rightarrow a^2 = 4x^2$

$\Rightarrow \therefore a = 2x$

So, the edge of the cube is $2x$.

i)When two cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x = 4x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

$\therefore$ Surface area of the cuboid = $2\,(lb + bh + lh)$

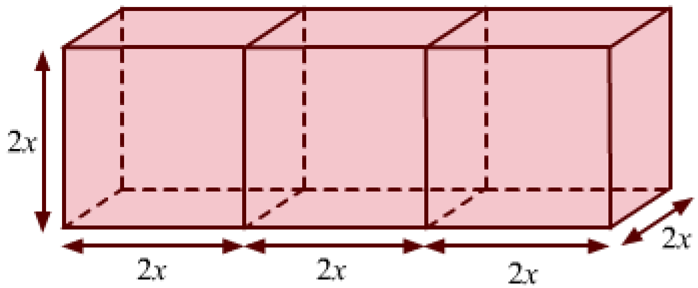
$= 2 \times (4x \times 2x + 2x \times 2x + 4x \times 2x)$

$= 2 \times (8x^2 + 4x^2 + 8x^2)$

$= 40x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $40x^2$.

ii)When three cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x + 2x = 6x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

∴ Surface area of the cuboid = $2(lb + bh + lh)$

$= 2 \times (6x \times 2x + 2x \times 2x + 6x \times 2x)$

$= 2 \times (12x^2 + 4x^2 + 12x^2)$

$= 56x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $56x^2$.

Hard

Example 1: The cost of flooring a twenty-metre-long room at Rs 5 per square metre is Rs 1000. If the cost of painting the four walls of the room at Rs 15 per square metre is Rs 1800, then find the height of the room.

Solution: The length (l) of the room is given as 20 m. Let b and h be its breadth and height respectively.

Area of the floor = $l \times b$

Cost of flooring at Rs 5 per m^2 = Rs 1000

So, $5 \times l \times b = 1000$

$$\Rightarrow 5 \times 20 \times b = 1000$$

$$\Rightarrow \therefore b = \frac{1000}{100} = 10$$

Area of the four walls = 2 ($bh + lh$)

Cost of painting the four walls at Rs 15 per m^2 = Rs 1800

So, 15 × [2 ($bh + lh$)] = 1800

⇒ 15 × [2 × (10 × h + 20 × h)] = 1800

$$\Rightarrow 30h = \frac{1800}{15 \times 2}$$
$$\Rightarrow \therefore h = \frac{1800}{15 \times 2 \times 30} = 2$$

Thus, the height of the room is 2 m.

Example 2: The internal measures of a cuboidal room are 20 m × 15 m × 12 m. Dinesh wants to paint the four walls of the room with orange colour and the roof of the room with white colour. 100 m^2 of surface can be painted using each can of orange paint and 125 m^2 of surface can be painted using each can of white paint. How many cans of each colour will be required? If the orange and white paints are available at Rs 250 per can and Rs 300 per can respectively, then how much money will be spent by Dinesh to paint the room?

Solution: Length (l) of the room = 20 m

Breadth (b) of the room = 15 m

Height (h) of the room = 12 m

Area of the room to be painted using orange colour= Area of the four walls of the room

= Lateral surface area of the room

= $2h$ ($l + b$)

= [2 × 12 (20 + 15)] m^2

= (24 × 35) m^2

= 840 m^2

It is given that 100 m^2 of surface can be painted using each can of orange paint.

∴ Number of cans of orange paint required $= \frac{\text{Area of the room painted using orange colour}}{\text{Area that can be painted using each can}}$

$= \frac{840}{100}$

= 8.4

= 9 (∵ 8 cans will be insufficient for the job)

Thus, 9 cans of orange paint will be required for painting the four walls of the room.

Area of the room to be painted using white colour= Area of the roof

$= l \times b$

$= (20 \times 15)\ m^2$

$= 300\ m^2$

It is given that $125\ m^2$ of surface can be painted using each can of white paint.

∴ Number of cans of white paint required $= \frac{\text{Area of the room painted using white colour}}{\text{Area that can be painted using each can}}$

$= \frac{300}{125}$

= 2.4

= 3 (∵ 2 cans will be insufficient for the job)

Thus, 3 cans of white paint will be required for painting the roof of the room.

Cost of each can of orange paint = Rs 250

⇒ Cost of 9 cans of orange paint = 9 × Rs 250 = Rs 2250

Cost of each can of white paint = Rs 300

⇒ Cost of 3 cans of white paint = 3 × Rs 300 = Rs 900

Thus, total money that will be spent in painting the room = Rs 2250 + Rs 900 = Rs 3150

Surface Area of Right Circular Cylinders

Surface Area of a Right Circular Cylinder

We come across many objects in our surroundings which are cylindrical, i.e., shaped like a **cylinder**, for example, pillars, rollers, water pipes, tube lights, cold-drink cans and LPG cylinders. This three-dimensional figure is found almost everywhere.

We can easily make cylindrical containers using metal sheets of any length and breadth. Say we have to make an open metallic cylinder (as shown below) of radius 14 cm and height 40 cm. How will we calculate the dimensions of the metal required for making this specific cylinder?

We will do so by calculating the surface area of the required cylinder. This surface area will be equal to the area of metal sheet required to make the cylinder.

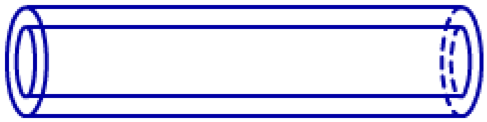
Knowledge of surface areas of three-dimensional figures is important in finding solutions to several real-life problems involving them. In this lesson, we will learn the formulae for the surface area of a right circular cylinder. We will also solve examples using these formulae.

Features of a right circular cylinder

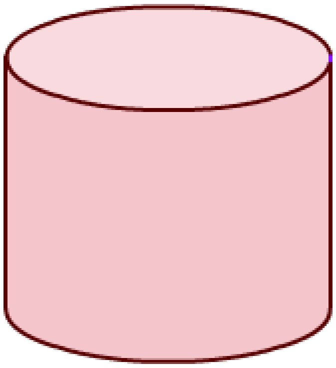
1. A right circular cylinder has two plane surfaces circular in shape.
2. The curved surface joining the plane surfaces is the lateral surface of the cylinder.
3. The two circular planes are parallel to each other and also congruent.
4. The line joining the centers of the circular planes is the axis of the cylinder.
5. All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
6. Radius of circular plane is the radius of the cylinder.

Two types of cylinders are given below.

1. Hollow cylinder: It is formed by the lateral surface only. Example: A pipe



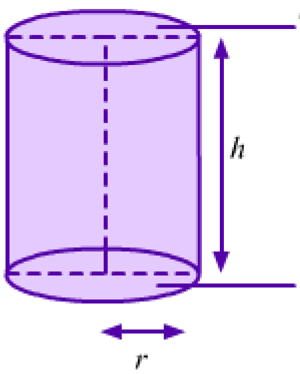
2. Solid cylinder: It is the region bounded by two circular plane surfaces with the lateral surface. Example: A garden roller



Solid cylinder

Formulae for the Surface Area of a Right Circular Cylinder

Consider a cylinder with base radius *r* and height *h*.



The formulae for the surface area of this cylinder are given as follows:

Curved surface area of the cylinder = 2π*rh*

Area of two circular faces of cylinder = 2π*r*²

Total surface area of the cylinder = 2π*r* (*r* + *h*)

Note: We take π as a constant and its value as $\frac{22}{7}$ or 3.14.

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the top and bottom surfaces. Total surface area refers to the sum of the areas of the top and bottom surfaces and the area of the curved surface.

Did You Know?

Pi

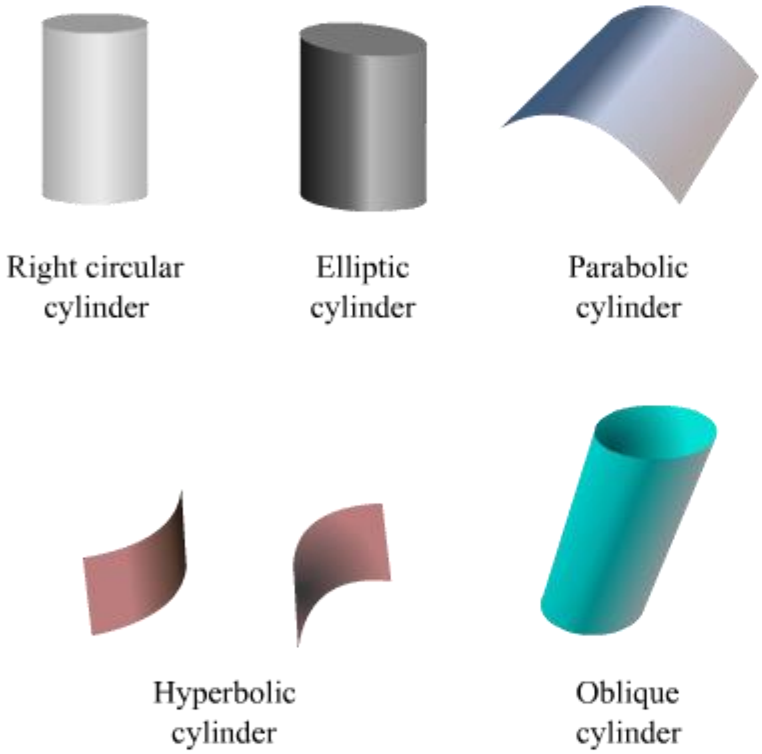
- Pi is a mathematical constant which is equal to the ratio of the circumference of a circle to its diameter.
- It is an irrational number represented by the Greek letter 'π' and its value is approximately equal to 3.14159.
- William Jones (1706) was the first to use the Greek letter to represent this number.
- Pi is also called 'Archimedes' constant' or 'Ludolph's constant'.
- Pi is a 'transcendental number', which means that it is not the solution of any finite polynomial with whole numbers as coefficients.
- Suppose a circle fits exactly inside a square; then, pi = $4\times\frac{\text{Area of the circle}}{\text{Area of the square}}$

Know Your Scientist

William Jones (1675–1749) was a Welsh mathematician who is primarily known for his proposal to use the Greek letter 'π' for representing the ratio of the circumference of a circle to its diameter. His book ***Synopsis Palmariorum Matheseos*** includes theorems on differential calculus and infinite series. In this book, π is used as an abbreviation for perimeter.

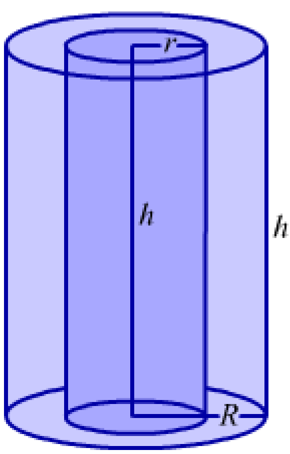
Whiz Kid

There are many types of cylinders—right circular cylinder (whose base is circular), elliptic cylinder (whose base is an ellipsis or oval), parabolic cylinder, hyperbolic cylinder, imaginary elliptic cylinder, oblique cylinder (whose top and bottom surfaces are displaced from each other), etc.



Formulae for the Surface Area of a Right Circular Hollow Cylinder

Consider a hollow cylinder of height *h* with external and internal radii *R* and *r* respectively,



Here, curved surface area, CSA = External surface area + Internal surface area

$$= 2\pi Rh + 2\pi rh$$
$$= 2\pi h\,(R + r)$$

Total surface area, TSA = Curved surface area + 2 × Base area

$$= 2\pi h\,(R + r) + 2 \times \left[\pi R^2 - \pi r^2\right]$$
$$= 2\pi h\,(R + r) + 2\pi\,(R + r)\,(R - r)$$
$$= 2\pi\,(R + r)\,(R - r + h)$$

Here, thickness of the hollow cylinder = $R - r$.

Solved Examples

Easy

Example 1: The curved surface area of a right circular cylinder of height 7 cm is 44 cm^2. Find the diameter of the base of the cylinder.

Solution:

Let r be the radius and h be the height of the cylinder.

It is given that:

h = 7 cm

Curved surface area of the cylinder = 44 cm^2

So, $2\pi rh$ = 44 cm^2

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 7 \text{ cm} = 44 \text{ cm}^2$$
$$\Rightarrow r = \frac{44 \times 7}{2 \times 22 \times 7} \text{ cm}$$
$$\Rightarrow \therefore r = 1 \text{ cm}$$

Thus, diameter of the base of the cylinder = $2r$ = 2 cm

Example 2: The radii of two right circular cylinders are in the ratio 4 : 5 and their heights are in the ratio 3 : 1. What is the ratio of their curved surface areas?

Solution:

Let the radii of the cylinders be $4r$ and $5r$ and their heights be $3h$ and h.

Let S_1 be the curved surface area of the cylinder of radius 4*r* and height 3*h*.

$\therefore S_1 = 2\pi \times 4r \times 3h = 24\pi rh$

Let S_2 be the curved surface area of the cylinder of radius 5*r* and height *h*.

$\therefore S_2 = 2\pi \times 5r \times h = 10\pi rh$

Now,

$$\frac{S_1}{S_2} = \frac{24\pi rh}{10\pi rh} = \frac{12}{5}$$
$$\Rightarrow S_1 : S_2 = 12 : 5$$

Thus, the curved surface areas of the two cylinders are in the ratio 12 : 5.

Medium

Example 1: Find the height and curved surface area of a cylinder whose radius is 14 dm and total surface area is 1760 dm².

Solution:

Radius (*r*) of the cylinder = 14 dm

Let the height of the cylinder be *h.*

Total surface area of the cylinder = 1760 dm^2

So, $2\pi r (r + h) = 1760\ dm^2$

$$\Rightarrow 2 \times \frac{22}{7} \times 14(14 + h)\text{dm} = 1760\ \text{dm}^2$$
$$\Rightarrow 14 + h = \frac{1760 \times 7}{2 \times 22 \times 14}\text{dm}$$
$$\Rightarrow 14 + h = 20\ \text{dm}$$
$$\Rightarrow \therefore h = (20 - 14)\text{dm} = 6\ \text{dm}$$

Thus, the height of the cylinder is 6 dm.

Now, curved surface area of the cylinder = $2\pi rh$

$$= (2 \times \frac{22}{7} \times 14 \times 6) \text{ dm}^2$$
$$= 528 \text{ dm}^2$$

Example 2: There are ten identical cylindrical pillars in a building. If the radius of each pillar is 35 cm and the height is 12 m, then find the cost of plastering the surface of all the pillars at the rate of Rs 15 per m^2.

Solution:

Radius (r) of one pillar $= 35 \text{ cm} = \frac{35}{100} \text{ m} = 0.35 \text{ m}$

Height (h) of one pillar = 12 m

∴ Curved surface area of one pillar = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 0.35 \times 12\right) \text{ m}^2$$
$$= 26.4 \text{ m}^2$$

⇒ Curved surface area of ten pillars = 10 × 26.4 m^2 = 264 m^2

Cost of plastering 1 m^2 of surface = Rs 15

⇒ Cost of plastering 264 m^2 of surface = Rs (15 × 264) = Rs 3960

Therefore, the cost of plastering the ten pillars of the building is Rs 3960.

Hard

Example 1: A cylindrical road roller is of diameter 175 cm and length 1.5 m. It has to cover an area of 0.33 hectare on the ground. How many complete revolutions must the roller take to cover the ground? (1 hectare = 10000 m^2)

Solution: Diameter of the cylindrical roller $= 175 \text{ cm} = \frac{175}{100} \text{ m} = \frac{7}{4} \text{ m}$

∴ Radius (r) of the cylindrical roller = $\frac{7}{8}$ m

Length (h) of the cylindrical roller = 1.5 m

Area covered by the roller in one complete revolution = Curved surface area of the roller

= $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times \frac{7}{8} \times 1.5\right) \text{ m}^2$$
$$= 8.25 \text{ m}^2$$

Area of the ground to be covered = 0.33 hectare = 0.33 × 10000 m^2 = 3300 m^2

∴ Number of complete revolutions $= \dfrac{\text{Area of the ground covered by the roller}}{\text{Area covered by the roller in one revolution}}$

$$= \frac{3300\text{m}^2}{8.25\text{m}^2}$$
$$= 400$$

Thus, the roller must take 400 complete revolutions to cover the ground.

Example 2: The internal diameter, thickness and height of a hollow cylinder are 20 cm, 1 cm and 25 cm respectively. What is the total surface area of the cylinder?

Solution: Internal diameter of the cylinder = 20 cm

∴ Internal radius (r) of the cylinder $= \dfrac{20}{2}\text{cm} = 10\,\text{cm}$

Thickness of the cylinder = 1 cm

∴ External radius (R) of the cylinder = (10 + 1) cm = 11 cm

Height (h) of the cylinder = 25 cm

Internal curved surface area of the cylinder = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 10 \times 25\right) \text{ cm}^2$$
$$= \frac{11000}{7}\text{cm}^2$$

External curved surface area of the cylinder = $2\pi Rh$

$$= \left(2 \times \frac{22}{7} \times 11 \times 25\right) \text{ cm}^2$$

$$= \frac{12100}{7} \text{cm}^2$$

The two bases of the cylinder are ring-shaped. Therefore, their area is given as follows:

Area of base = $\pi (R^2 - r^2)$

$$= \left[\frac{22}{7}\left(11^2 - 10^2\right)\right] \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 21\right) \text{ cm}^2$$

$$= 66 \text{ cm}^2$$

So, total surface area of the cylinder = Internal CSA + External CSA + 2 × Area of base

$$= \left(\frac{11000}{7} + \frac{12100}{7} + 2 \times 66\right) \text{ cm}^2$$

$$= \left(\frac{23100}{7} + 132\right) \text{ cm}^2$$

$$= (3300 + 132) \text{ cm}^2$$

$$= 3432 \text{ cm}^2$$

Surface Areas of Cones

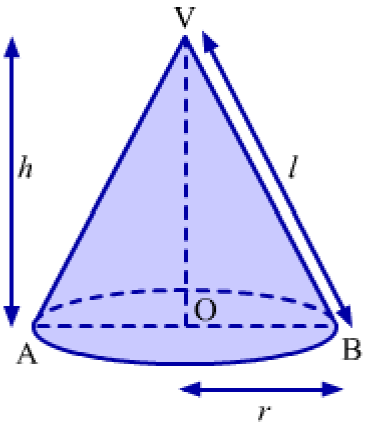
Surface Area of a Right Circular Cone

Traffic cones, conical tents, party hats, ice cream cones are some examples of objects shaped like a **cone**. The knowledge of the surface area of a cone is essential in the manufacture of such conical objects. Take, for example, the following case.

X Ltd. is a company that organizes adventures trips. It has a contract with Y Ltd., a company that manufactures tents. Y Ltd. uses canvas to make the specific conical tents ordered by X Ltd. Now, the area of canvas required to make one such conical tent is exactly equal to the surface area of the conical tent. Thus, Y Ltd is able to order the required amount of canvas from the market to prepare the tents according to the specifications.

This is just one of the many examples from real life involving the concept of surface area. In this lesson, we will learn the formulae for the surface area of a right circular cone. We will also apply the formulae in solving a few examples.

Formulae for the Surface Area of a Right Circular Cone



Consider a cone with a base radius *r*, height *h* and slant height *l*.

The fixed point V is the **vertex** of the cone and the fixed line VO is the **axis** of the cone.

The length of line segment joining the vertex to the centre O of the base is called the **height of the base** and the length of the line segment joining the vertex to any point on the circular edge of the base is called the **slant height** of the cone.

The relation between the height, radius and slant height of the cone is: $l^2 = r^2 + h^2$.

The formulae for the surface area of the given cone are given as follows:

Curved surface area of the cone = πrl

Total surface area of the cone = $\pi r (l + r)$

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the base, and total surface area refers to the sum of the area of the base and the area of the curved surface.

Did You Know? A cone is the shape obtained by rotating a right triangle around one of its two shorter sides.

Did You Know? A cone is a three-dimensional geometric figure that does not have uniform or congruent cross-sections.

Largest Cone Cut Out from a Cylinder

Solved Examples

Easy

Example 1: The curved surface area of a cone is 1914 cm² and its base radius is 21 cm. Find

i)the slant height of the cone.

ii)the total surface area of the cone.

Solution:

i)Radius (r) of the cone = 21 cm

Curved surface area of the cone = 1914 cm^2

Let the slant height of the cone be l.

$\therefore \pi rl = 1914 \text{ cm}^2$

$$\Rightarrow \left(\frac{22}{7} \times 21 \times l\right) = 1914$$

$$\Rightarrow l = \frac{1914}{66} \text{cm}$$

$$\Rightarrow \therefore l = 29 \text{ cm}$$

Thus, the slant height of the cone is 29 cm.

ii)Total surface area of the cone = $\pi r\,(l + r)$

$$= \left[\frac{22}{7} \times 21 \times (29 + 21)\right] \text{ cm}^2$$

$$= \frac{22}{7} \times 21 \times 50 \text{ cm}^2$$

$$= 3300 \text{ cm}^2$$

Example 2: The total surface area of a cone is 33264 cm^2 and its base radius and slant height are in the ratio 3 : 5. Find the slant height of the cone.

Solution: Let the radius and slant height of the cone be $3x$ and $5x$ respectively.

Total surface area of the cone = 33264 cm^2

$$\Rightarrow \pi r(l + r) = 33264 \text{cm}^2$$

$$\Rightarrow \frac{22}{7} \times 3x(5x + 3x) = 33264 \text{cm}^2$$

$$\Rightarrow 24x^2 = \frac{33264 \times 7}{22} \text{ cm}^2$$

$$\Rightarrow x^2 = \frac{33264 \times 7}{22 \times 24} \text{cm}^2$$

$$\Rightarrow x^2 = 441 \text{ cm}^2$$

$$\Rightarrow \therefore x = \sqrt{441} \text{ cm} = 21 \text{ cm}$$

So, slant height of the cone = $5x$ = 5 × 21 cm = 105 cm

Medium

Example 1: The height and radius of the base of a conical tomb are 8 m and 6 m respectively. Find the cost of whitewashing the outer surface of the tomb at the rate of Rs 2000 per 50 m^2.

Solution:

Radius (r) of the conical tomb = 6 m

Height (h) of the base of the conical tomb = 8 m

Let the slant height of the conical tomb be l.

We know that $l^2 = r^2 + h^2$

$\Rightarrow l^2 = (6^2 + 8^2)$ m^2

$\Rightarrow l^2 = (36 + 64)$ m^2

$\Rightarrow l^2 = 100$ m^2

$\Rightarrow \therefore l = \sqrt{100}$ m $= 10$ m

∴ Curved surface area of the conical tomb = πrl

$= \left(\frac{22}{7} \times 6 \times 10\right)$ m^2

$= 188.57$ m^2

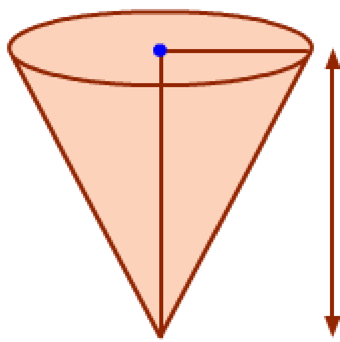
Cost of whitewashing 50 m^2 of surface = Rs 2000

⇒ Cost of whitewashing 1 m^2 of surface = Rs $\frac{2000}{50}$ = Rs 40

⇒ Cost of whitewashing 188.57 m^2 of surface = 188.57 × Rs 40 = Rs 7542.80

Thus, the cost of whitewashing the outer surface of the tomb is Rs 7542.80.

Example 2: A corncob (which is shaped like a cone) is of length 30 cm and the radius of its broadest end is 4 cm. If about 5 grains are present per square centimetre of the cob, then approximately how many grains are there on the entire cob?



Solution:

Total grains on the cob = Curved surface area of the cob × Number of grains per cm^2

Radius (r) of the cob = 4 cm

Height (h) of the base of the cob = 30 cm

Let the slant height of the cob be l.

We know that $l^2 = r^2 + h^2$

$\Rightarrow l^2 = (4^2 + 30^2)\ cm^2$

$\Rightarrow l^2 = (16 + 900)\ cm^2$

$\Rightarrow l^2 = 916\ cm^2$

$\Rightarrow \therefore l = \sqrt{916}\ cm = 30.26\ cm$

Curved surface area of the cob = $\pi r l$

$$= \left(\frac{22}{7} \times 4 \times 30.26\right)\ cm^2$$
$$= 380.4\ cm^2$$

Total number of grains = 380.4 × 5 = 1902

Thus, there are about 1902 grains on the entire cob.

Hard

Example 1: A cone and a cylinder have the same radius and height. If the ratio of the radius to height is 5 : 12, then find the ratio of the curved surface area of the cone to that of the cylinder.

Solution:

The cylinder and the cone have the same radius and height. Let *r* be this radius and *h* be the height.

It is given that:

$$\frac{r}{h}=\frac{5}{12}$$

⇒ $r = 5x$ and $h = 12x$, where x is any constant

Slant height (l) of the cone $=\sqrt{r^2+h^2}$

$$=\sqrt{(5x)^2+(12x)^2}$$
$$=\sqrt{25x^2+144x^2}$$
$$=\sqrt{169}x$$
$$=13x$$

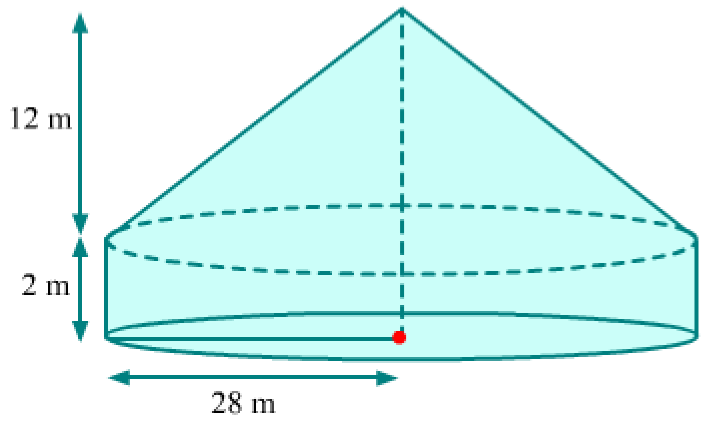
$$\therefore \frac{\text{Curved surface area of the cone}}{\text{Curved surface area of the cylinder}}=\frac{\pi rl}{2\pi rh}$$

$$=\frac{13x}{2\times 12x}$$
$$=\frac{13}{24}$$

Hence, the curved surface areas of the cone and the cylinder are in the ratio 13 : 24.

Example 2: A cylindrical tent of height 2 m and radius 28 m is surmounted by a right circular cone. If the total height of the tent is 14 m and the cost of papering is Rs 3 per square metre, then calculate the total money spent in papering the inner side of the tent.

Solution:



Radius (r) of the cylindrical part = 28 m

Height (h) of the cylindrical part = 2 m

∴ Curved surface area of the cylindrical part = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 28 \times 2\right) \text{ m}^2$$
$$= 352 \text{ m}^2$$

Radius of the base of the conical part = Radius of the cylindrical part = 28 m

Let H and l be respectively the height and slant height of the conical part.

H = (14 − 2) m = 12 m (âˆµ Total height of the tent = 14 m)

We know that $l^2 = r^2 + H^2$

$\Rightarrow l^2 = (28^2 + 12^2)$ m^2

$\Rightarrow l^2 = (784 + 144)$ m^2

$\Rightarrow l^2 = 928$ m^2

$$\Rightarrow \therefore l = \sqrt{928} \text{ m} = 30.46 \text{ m}$$

∴ Curved surface area of the conical part = πrl

$$= \frac{22}{7} \times 28 \times 30.46 \text{ m}^2$$
$$= 2680.48 \text{ m}^2$$

Total surface area = Sum of the curved surface areas of the cylindrical and conical parts

= (352 + 2680.48) m^2

= 3032.48 m^2

Cost of papering 1 m^2 of surface = Rs 3

⇒ Cost of papering 3032.48 m^2 of surface = Rs 3 × 3032.48 = Rs 9097.44

Thus, the total money spent in papering the inner side of the tent is Rs 9097.44.

Surface Areas of Sphere and Hemisphere

Surface Areas of a Sphere and a Hemisphere

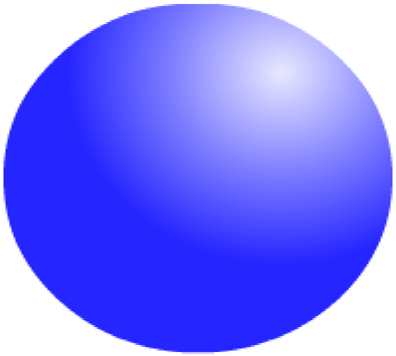
What images come to your mind when the word '**sphere**' is mentioned? The light hollow ball used in table tennis, the leather ball used in cricket, the inflatable balls used in the games of football and basketball and the heavy metallic shots used for shot-putting are all examples of the perfectly round three-dimensional shape called sphere. Now, if you were to cut each of the objects mentioned above along its diameter, then you would obtain the three-dimensional figure called hemisphere. As its name indicates, a hemisphere is the half of a sphere. Any sphere when cut along the diameter yields two equal hemispheres.

A sphere has only a curved surface; so, in its case, the total surface area is the same as the area of its curved surface. This, however, is not the case with a hemisphere. Consider the whole watermelon and the half of the same shown below.

Clearly, the whole watermelon has only a curved exterior, but what about its half? Observe how the half of the watermelon has both a curved exterior and a flat surface. So, in case of a hemisphere the total surface area is different from the area of its curved surface.

Sphere and hemisphere

A **sphere** is a solid described by the rotation of a semi-circle about a fixed diameter.

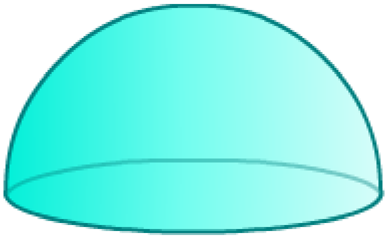


Sphere

Properties of a sphere:

1. A sphere has a centre.
2. All the points on the surface of the sphere are equidistant from the centre.
3. The distance between the centre and any point on the surface of the sphere is the radius of the sphere.

Hemisphere:A plane through the centre of the sphere divides it into two equal parts each is called a hemisphere.

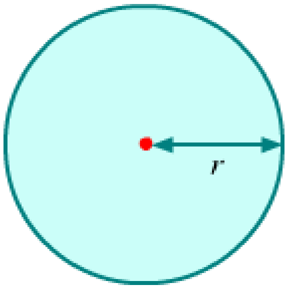


Hemisphere

In this lesson, we will learn the formulae for the surface areas of a sphere and a hemisphere. We will also solve problems using the same.

Formula for the Surface Area of a Sphere

Consider a sphere of radius *r*.



The formula for the surface area (curved or total) of this sphere is given as follows:

Surface area of a sphere = $4\pi r^2$

As mentioned before, the total surface area of a sphere is the same as its curved surface area since a sphere has only a curved surface.

Did You Know?

Among all geometric shapes, a sphere has the smallest surface area for a given volume. Take, for example, bubbles and water droplets. Their spherical shape enables them to hold as much air as possible with the least surface area.

Solved Examples

Easy

Example 1: **What is the radius of a globe whose surface area is 1256 cm^2? (Use π = 3.14)**

Solution: Let the radius of the globe be r.

Surface area of a sphere = $4\pi r^2$

It is given that the surface area of the globe is 1256 cm^2.

So, $4\pi r^2 = 1256\ cm^2$

$\Rightarrow 4 \times 3.14 \times r^2 = 1256\ cm^2$

$\Rightarrow 12.56 \times r^2 = 1256\ cm^2$

$$\Rightarrow r^2 = \left(\frac{1256}{12.56}\right)\ cm^2$$
$$\Rightarrow r^2 = 100\ cm^2$$
$$\Rightarrow \therefore r = \left(\sqrt{100}\right)\ cm = 10\ cm$$

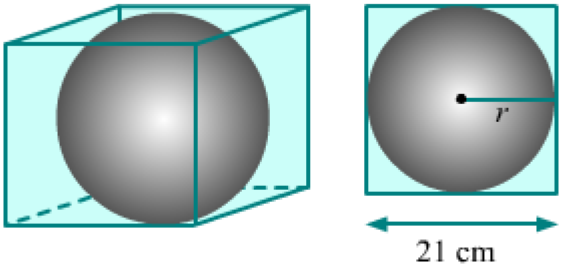
Thus, the radius of the globe is 10 cm.

Medium

Example 1: **Find the surface area of the largest sphere that can be inscribed in a cube of edge 21 cm.**

Solution: Edge of the cube = 21 cm

Suppose the largest sphere that can be inscribed in this cube has a radius *r*.



This sphere will touch all the six walls of the cube. Therefore, the diameter of the sphere will be equal to the edge of the cube.

So, $2r = 21$ cm

$$\Rightarrow r = \left(\frac{21}{2}\right) \text{ cm}$$

Now, surface area of the required sphere = $4\pi r^2$

$$= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{ cm}^2$$
$$= 1386 \text{ cm}^2$$

Example 2: The diameter of a football is approximately $\frac{20}{19}$ times the diameter of a volleyball. What is the ratio of their surface areas?

Solution: Let the diameter of the volleyball be *x*.

∴ Diameter of the football = $\frac{20}{19}x$

Now, radius (r_1) of the volleyball $= \frac{x}{2}$

Radius (r_2) of the football $= \frac{10}{19}x$

Required ratio $= \dfrac{\text{Surface area of the football}}{\text{Surface area of the volleyball}}$

$$= \frac{4\pi r_2^2}{4\pi r_1^2}$$

$$= \left(\frac{r_2}{r_1}\right)^2$$

$$= \left(\frac{\frac{10}{19}x}{\frac{x}{2}}\right)^2$$

$$= \left(\frac{20}{19}\right)^2$$

$$= \frac{400}{361}$$

Thus, the surface areas of the football and the volleyball are in the ratio 400 : 361.

Hard

Example 1: **If the diameter of a sphere is increased by 25%, then what will be the percentage increase in its curved surface area?**

Solution: Let *r* be the radius and *S* be the curved surface area of the sphere.

$\therefore S = 4\pi r^2$

Percentage increase in diameter = 25

∴ Increase in diameter = 25% of 2*r* = $\left(\frac{25}{100} \times 2r\right) = \frac{r}{2}$

⇒ Increased diameter = $2r + \frac{r}{2} = \frac{5r}{2}$

∴ Increased radius = $\frac{5r}{4}$

Let *S* ' be the new curved surface area of the sphere.

$$\therefore S' = 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$$

Now, increase in curved surface area $= S' - S$

$$= \frac{25\pi r^2}{4} - 4\pi r^2$$
$$= \frac{9\pi r^2}{4}$$

$\therefore$ Percentage increase in curved surface area $= \frac{S' - S}{S} \times 100$

$$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100$$
$$= \frac{900}{16}$$
$$= 56.25$$

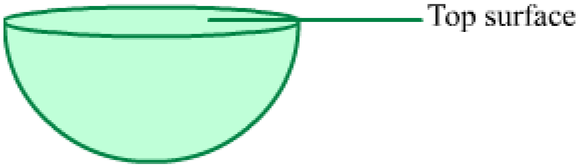
Thus, the curved surface area of the sphere will increase by 56.25%.

Formulae for the Surface Area of a Hemisphere

A hemisphere is a three dimensional solid having two faces, one edge and no vertex.

Since a hemisphere is obtained by cutting a sphere along its diameter, the radius of a hemisphere is the same as that of the sphere from which it is cut.

Consider a hemisphere with radius *r*.



The formulae for the surface area of this hemisphere are given as follows:

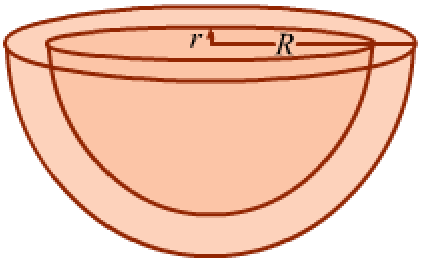
Curved surface area of the hemisphere = $2\pi r^2$

Total surface area of the hemisphere = $3\pi r^2$

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the area of the top surface, and total surface area refers to the sum of the area of the curved surface and the area of the top surface.

Formula for the Surface Area of a Hollow Hemisphere Sphere

Let *R* and *r* be the outer and inner radii of the hollow hemisphere.



Here, curved surface area = Outer surface area + Inner surface area

$$= 2\pi R^2 + 2\pi r^2$$
$$= 2\pi \left(R^2 + r^2\right)$$

The total surface area = curved surface area + Area at the base

$$= 2\pi \left(R^2 + r^2\right) + \pi \left(R^2 - r^2\right)$$
$$= \pi \left(2R^2 + 2r^2 + R^2 - r^2\right)$$
$$= \pi \left(3R^2 + r^2\right)$$

Solved Examples

Easy

Example 1: **If the total surface area of a hemisphere is 462 cm^2, then find its radius.**

Solution: Let the radius of the hemisphere be *r*.

Total surface area of the hemisphere is given by the formula $3\pi r^2$.

It is given that the total surface area of the hemisphere is 462 cm^2.

So, 462 cm^2 = $3\pi r^2$

$$\Rightarrow 462\ \text{cm}^2 = 3\times\frac{22}{7}\times r^2$$

$$\Rightarrow r^2 = \frac{462\times 7}{3\times 22}\text{cm}^2$$

$$\Rightarrow r^2 = 49\ \text{cm}^2$$

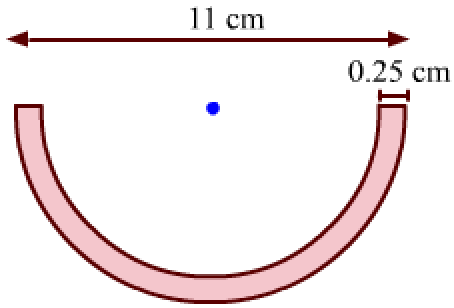
$$\Rightarrow \therefore r = \sqrt{49}\ \text{cm} = 7\ \text{cm}$$

Thus, the radius of the hemisphere is 7 cm.

Medium

Example 1: **A hemispherical steel bowl is 0.25 cm thick. The outer diameter of the bowl is 11 cm. Calculate the cost of tin-plating the inner surface of the bowl at the rate of Rs 16 per**
100 cm^2.

Solution: The figure according to the given specifications can be made as follows:



Outer diameter of the hemispherical bowl = 11 cm

∴ Outer radius of the hemispherical bowl = $\frac{11}{2}\ \text{cm} = 5.5\ \text{cm}$

Thickness of the hemispherical bowl = 0.25 cm

∴ Inner radius (r) of the hemispherical bowl = (5.5 − 0.25) cm = 5.25 cm

Inner curved surface area of the hemispherical bowl **=** $2\pi r^2$

$$= (2\times\frac{22}{7}\times 5.25\times 5.25)\ \text{cm}^2$$

$$= 173.25\ \text{cm}^2$$

Cost of tin-plating 100 cm^2 of surface = Rs 16

∴ Cost of tin-plating 1 cm^2 of surface = $\text{Rs}\frac{16}{100}$

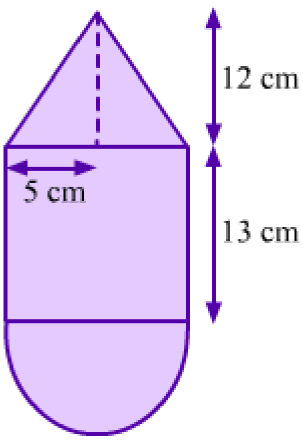
⇒ Cost of tin-plating 173.25 cm^2 of surface = $\text{Rs}\left(173.25\times\frac{16}{100}\right)=\text{Rs } 27.72\approx\text{Rs } 27.80$

Thus, the cost of tin-plating the inner surface of the hemispherical bowl is Rs 27.80.

Hard

Example 1: **A toy is in the shape of a right circular cylinder with a hemisphere at one end and a cone at the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. Calculate the curved surface area of the toy if the height of the conical part is 12 cm.**

Solution: The figure according to the given specifications can be made as follows:



Radius (r) of the cylinder = 5 cm

Height (H) of the cylinder = 13 cm

Height (h) of cone = 12 cm

Radii of the cone and the hemisphere = Radius of the cylinder = 5 cm

Now, slant height (l) of the cone $=\sqrt{r^2+h^2}$

$= \sqrt{5^2 + 12^2}$ cm

$= \sqrt{25 + 144}$ cm

$= \sqrt{169}$ cm

$= 13$ cm

∴ Surface area of the toy = CSA of the hemisphere + CSA of the cylinder + CSA of the cone

$= 2\pi r^2 + 2\pi rH + \pi rl$

$= \pi r(2r + 2H + l)$

$= \frac{22}{7} \times 5 \times (2 \times 5 + 2 \times 13 + 13)$ cm^2

$= \frac{22}{7} \times 5 \times 49$ cm^2

$= 770$ cm^2

Thus, the curved surface area of the toy is 770 cm^2.

Volumes of Cubes and Cuboids

Volumes of a Cube and a Cuboid

Abhinav's mother gives him a container, asking him to go to the neighbouring milk booth and buy 2.5 L of milk. What does '2.5 L'represent? It represents the amount of milk that Abhinav needs to buy. In other words, it is the volume of milk that is to be bought.

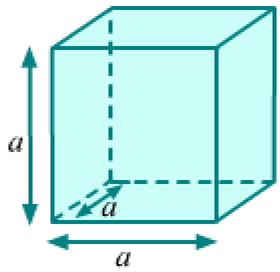
After buying the milk, Abhinav notices that the container is full up to its brim. He says to himself, 'This container has no capacity to hold any more milk.' What does the word 'capacity' indicate? **The space occupied by a substance is called its volume. The capacity of a container is the volume of a substance that can fill the container completely.** In this case, the volume and the capacity of the container are the same. The standard units which are used to measure the volume are **cm^3 (cubic centimetre)** and **m^3 (cubic metre).**

In this lesson, we will learn the formulae for the volumes or capacities of cubic and cuboidal objects. We will also solve examples using these formulae.

Did You Know?

A cube is one among the five platonic solids. This means that it is a regular and convex polyhedron with the same number of faces meeting at each vertex.

Formulae for the Volumes of a Cube and a Cuboid

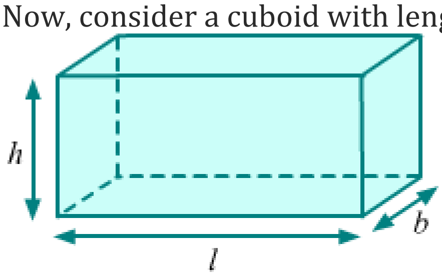


Consider a cube with an edge *a*.

The formula for the volume of this cube is given as follows:

Volume of the cube = a^3

Now, consider a cuboid with length *l*, breadth *b* and height *h*.



The formula for the volume of this cuboid is given as follows:

Volume of the cuboid = $l \times b \times h$

Concept Builder

The units of capacity and volume are interrelated as follows:

- $1\ cm^3 = 1$ mL
- $1000\ cm^3 = 1$ L

- $1\ m^3 = 1\ kL = 1000\ L$

Did You Know?

- A cube has the maximum volume among all cuboids with equal surface area.
- A cube has the minimum surface area among all cuboids with equal volume.

Solved Examples

Easy

Example 1: Find the volumes of cubes of given sides.

(a) 2 cm (b) 5 m (c) 12 cm (d) 15 m

Solution: (a)

Measure of side of cube = 2 cm

Volume of cube = $(\text{Side})^3 = 2^3\ cm^3 = 8\ cm^3$

(b)

Measure of side of cube = 5 m

Volume of cube = $(\text{Side})^3 = 5^3\ m^3 = 125\ m^3$

(c)

Measure of side of cube = 12 cm

Volume of cube = $(\text{Side})^3 = 12^3\ cm^3 = 1728\ cm^3$

(d)

Measure of side of cube = 15 m

Volume of cube = $(\text{Side})^3 = 15^3\ m^3 = 3375\ m^3$

Example 2: Find the volumes of cuboids of given dimensions.

(a) length = 5 cm, breadth = 2 cm, height = 6 cm

(b) length = 15 cm, breadth = 10 cm, height = 30 cm

(c) length = 1 m, breadth = 0.5 m, height = 1.5 m

Solution: (a)

We have

length = 5 cm, breadth = 2 cm, height = 6 cm

∴ Volume of cuboid = length × breadth × height

$= (5 \times 2 \times 6)\ cm^3$

$= 60\ cm^3$

(b)

We have

length = 15 cm, breadth = 10 cm, height = 30 cm

∴ Volume of cuboid = length × breadth × height

$= (15 \times 10 \times 30)\ cm^3$

$= 4500\ cm^3$

(c)

We have

length = 1 m, breadth = 0.5 m, height = 1.5 m

∴ Volume of cuboid = length × breadth × height

$= (1 \times 0.5 \times 1.5)\ m^3$

$= 0.75\ m^3$

Example 3: If a cubical tank can contain 1331000 L of water, then find the edge of the tank.

Solution: Capacity of the cubical tank = 1331000 L

$= 1331\ m^3$ (∵ $1000\ L = 1\ m^3$)

Now, capacity of the tank = Volume of water that can be contained in the tank

We know that volume of water in the tank = $(\text{Edge})^3$

$\Rightarrow (\text{Edge})^3 = 1331 \text{ m}^3$

$\Rightarrow \therefore \text{Edge} = 11 \text{ m}$

Thus, the edge of the cubical tank is 11 m.

Example 4: Find the height of the cuboid whose volume is 840 cm³and the area of whose base is
120 cm².

Solution: Let the length, breadth and height of the cuboid be *l*, *b* and *h* respectively.

Area of the base of the cuboid = 120 cm^2

$\therefore l \times b = 120 \text{ cm}^2$

Volume of the cuboid = 840 cm^3

$\therefore l \times b \times h = 840 \text{ cm}^3$

$\Rightarrow 120 \text{ cm}^2 \times h = 840 \text{ cm}^3$ ($\because l \times b = 120 \text{ cm}^2$)

$$\Rightarrow h = \frac{840}{120} \text{ cm}$$
$$\Rightarrow \therefore h = 7 \text{ cm}$$

Thus, the height of the cuboid is 7 cm.

Example 5: If the ratio of the edges of two cubes is 2 : 5, then find the ratio of their volumes.

Solution: Let the edges of the cubes be $a = 2x$ and $b = 5x$.

Ratio of the volumes of the cubes $= \dfrac{\text{Volume of the first cube}}{\text{Volume of the second cube}}$

$$= \frac{a^3}{b^3}$$

$$= \frac{(2x)^3}{(5x)^3}$$

$$= \frac{8x^3}{125x^3}$$

$$= \frac{8}{125}$$

Thus, the volumes of the cubes are in the ratio 8 : 125.

Medium

Example 1: **A solid cube of edge 18 cm is cut into eight cubes of equal volume. Find the dimension of each new cube. Also find the ratio of the total surface area of the bigger cube to that of the new cubes formed.**

Solution: Let the edge of each new cube be *x*.

According to the question, we have:

Volumes of 8 cubes each of edge *x* = Volume of cube of edge 18 cm

$$\Rightarrow 8 \times x^3 = (18 \text{ cm})^3$$

$$\Rightarrow x^3 = \frac{18 \text{ cm} \times 18 \text{ cm} \times 18 \text{ cm}}{8} = 729 \text{ cm}^3$$

$$\Rightarrow x^3 = (9 \text{ cm})^3$$

$$\Rightarrow \therefore x = 9 \text{ cm}$$

Thus, the edge of each new cube is 9 cm.

Total surface area (S_1) of the bigger cube = 6 × (18 cm)^2

Total surface area of 8 cubes (S_2) each of edge 9 cm = 8 × [6 × (9 cm)^2]

$$\therefore \frac{S_1}{S_2} = \frac{6 \times 18^2}{8 \times 6 \times 9^2} = \frac{1}{2}$$

Hence, the required ratio is 1 : 2.

Example 2: A hostel having strength of 300 students requires on an average 36000 L of water per day. It has a tank measuring 10 m × 8 m × 9 m. For how many days will the water in the tank filled to capacity last?

Solution: Let the cuboidal tank have length l, breadth b and height h.

It is given that $l = 10$ m, $b = 8$ m and $h = 9$ m.

Capacity of the tank = $l \times b \times h$ = 10 m × 8 m × 9 m = 720 m^3

∴ Amount of water in the tank filled to capacity = 720 m^3 = 720000 L (∵ 1000 L = 1 m^3)

Amount of water used by 300 students in 1 day = 36000 L

Number of days for which the water in the full tank will last $= \dfrac{\text{Amount of water in the full tank}}{\text{Amount of water used in a day}}$

$$= \frac{720000}{36000}$$
$$= 20$$

Thus, the water in the tank filled to capacity will last for 20 days.

Example 3: The dimensions of a wall in a godown are 25 m × 0.3 m × 10 m. How many bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall?

Solution: Length (L) of the wall = 25 m = (25 × 100) cm = 2500 cm

Breadth (B) of the wall = 0.3 m = (0.3 × 100) cm = 30 cm

Height (H) of the wall = 10 m = (10 × 100) cm = 1000 cm

∴ Volume of the wall = $L \times B \times H$ = (2500 × 30 × 1000) cm^3

Length (l) of one brick = 25 cm

Breadth (b) of one brick = 10 cm

Height (h) of one brick = 5 cm

∴ Volume of one brick = $l \times b \times h$ = (25 × 10 × 5) cm^3

Number of bricks used to construct the wall $= \dfrac{\text{Volume of the wall}}{\text{Volume of one brick}}$

$= \dfrac{2500 \times 30 \times 1000}{25 \times 10 \times 5}$
$= 60000$

Thus, 60000 bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall.

Example 4: A storeroom is in the form of a cuboid with dimensions 90 m × 150 m × 120 m. How many cubical boxes of edge 60 dm can be stored in the room?

Solution: Length (l) of the storeroom = 90 m

Breadth (b) of the storeroom = 150 m

Height (h) of the storeroom = 120 m

∴ Volume of the storeroom = $l \times b \times h = (90 \times 150 \times 120)\ \text{m}^3$

Edge (a) of one cubical box $= 60\ \text{dm} = \left(\dfrac{60}{10}\right)\ \text{m} = 6\ \text{m}$

∴ Volume of one box = $a^3 = (6)^3\ \text{m}^3$

Number of boxes that can be stored in the room $= \dfrac{\text{Volume of the storeroom}}{\text{Volume of one box}}$

$= \dfrac{90 \times 150 \times 120}{6 \times 6 \times 6}$
$= 7500$

Thus, 7500 cubical boxes of edge 60 dm can be stored in the room.

Hard

Example 1: A man-made canal is 5 m deep and 60 m wide. The water in the canal flows at the rate of
3 km/h. The canal empties its water into a reservoir. How much water will fall into the reservoir in 10 minutes?

Solution: Depth (h) of the canal = 5 m

Width (b) of the canal = 60 m

Length (l) of the canal is the rate of water flowing per hour = 3 km = 3000 m

Amount of water flowing per hour = $l \times b \times h$ = $(3000 \times 60 \times 5)$ m^3 = 900000 m^3 = 900000 kL ($\because$ 1 m^3 = 1 kL)

$\therefore$ Amount of water flowing in 60 min = 900000 kL

$\Rightarrow$ Amount of water flowing in 1 minute $= \left(\frac{900000}{60}\right) \text{kL}$

$\Rightarrow$ Amount of water flowing in 10 minutes $= \left(\frac{900000}{60} \times 10\right) \text{kL}$

= 150000 kL

Thus, 150000 kL of water will fall into the reservoir in 10 minutes.

Example 2: **The external length, breadth and height of a closed rectangular wooden box are 9 cm, 5 cm and 3 cm respectively. The thickness of the wood used is 0.25 cm. The box weighs 7.5 kg when empty and 50 kg when it is filled with sand. Find the weights of one cubic cm of wood and one cubic cm of sand.**

Solution: External length (L) of the wooden box = 9 cm

External breadth (B) of the wooden box = 5 cm

External height (H) of the wooden box = 3 cm

$\therefore$ External volume of the wooden box = $L \times B \times H$ = $(9 \times 5 \times 3)$ cm^3 = 135 cm^3

Thickness of the wood used = 0.25 cm

Internal length (l) of the wooden box = 9 cm − (0.25 cm + 0.25 cm) = 8.5 cm

Internal breadth (b) of the wooden box = 5 cm − (0.25 cm + 0.25 cm) = 4.5 cm

Internal height (h) of the wooden box = 3 cm − (0.25 cm + 0.25 cm) = 2.5 cm

$\therefore$ Internal volume of the wooden box = $l \times b \times h$ = $(8.5 \times 4.5 \times 2.5)$ cm^3 = 95.625 cm^3

Now, volume of the wood = External volume of the box − Internal volume of the box

= (135 – 95.625) cm^3

= 39.375 cm^3

Weight of the empty box = 7.5 kg

⇒ Weight 39.375 cm^3 of wood = 7.5 kg

∴ Weight of 1 cm^3 of wood $= \left(\frac{7.5}{39.375}\right) \text{kg} = 0.19 \text{ kg}$

Now, volume of sand = Internal volume of the box = 95.625 cm^3

Weight of sand = Weight of the box filled with sand – Weight of the empty box

= (50 – 7.5) Kg

= 42.5 Kg

⇒ Weight of 95.625 cm^3 of sand = 42.5 kg

∴ Weight of 1 cm^3 of sand $= \left(\frac{42.5}{95.625}\right) \text{kg} = 0.44 \text{ kg}$

Volume of Right Circular Cylinders

Water tanks like the ones shown below are a common enough sight.

Clearly, these tanks are cylindrical or shaped like a cylinder. The choice of this shape for a water tank (and many other storage containers) is because a cylinder provides a large volume for a little surface area. Also, this shape can withstand much more pressure than a cube or a cuboid, which makes it easy to transport. Another example of a cylindrical storage container is the LPG cylinder.

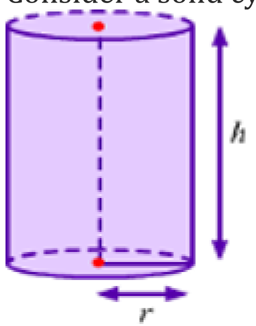
The amount of space occupied by a water tank is the same as the volume of the tank. So, to find the capacity or the amount of space occupied by a tank, we need to find the volume of the tank. In this lesson, we will learn the formula to calculate the volume of a right circular cylinder and solve some examples using the same.

Did You Know?

LPG tanks are cylinder-shaped so that they can withstand the pressure inside them. If these tanks were square or rectangular in shape, then an increase in pressure inside them would cause the tanks to reform themselves so as to gain a rounded shape. This, in turn, could result in leakage at the corners. Actual LPG tanks are designed to have no corners.

Formula for the Volume of a Right Circular Cylinder

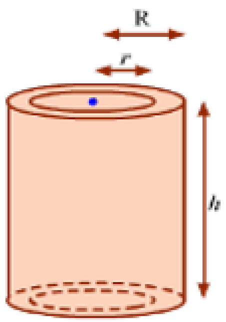
Consider a solid cylinder with r as the radius of the circular base and h as the height.



The formula for the volume of this right circular solid cylinder is given as follows:

Volume of the solid cylinder = Area of base × Height

Volume of the solid cylinder = $\pi r^2 h$



Consider a hollow cylinder with internal and external radii as r and R respectively, and height as h.

The formula for the volume of this right circular hollow cylinder is given as follows:

Volume of the hollow cylinder = π ($R^2 - r^2$) h

In right prisms, top and base surfaces are congruent and parallel while lateral faces are perpendicular to the base. Thus, their volumes can also be calculated in the same manner as that of right cylinders.

Volume of the right prism = Area of base × Height

Did You Know? The volume of a pizza (which is always cylindrical in shape) is hidden in its name itself. If we take the radius of a pizza to be 'z' and its thickness to be 'a', then its volume is '$\pi z^2 a$' or '**pi.z.z.a**'.

Solved Examples

Easy

Example 1: A cylindrical tank can hold 11000 L of water. What is the radius of the base of the tank if its height is 3.5 m?

Solution: Let r be the radius of the base of the cylindrical tank.

Height (h) of the tank = 3.5 m

Volume of the tank = 11000 L = 11 m^3 (∵ 1000 L = 1 m^3)

Volume of a cylinder = $\pi r^2 h$

In this case, we have

$$\pi r^2 h = 11 \text{ m}^3$$
$$\Rightarrow \left(\frac{22}{7} \times r^2 \times 3.5 \text{ m}\right) = 11 \text{ m}^3$$
$$\Rightarrow 11\, r^2 = 11 \text{ m}^2$$

$\Rightarrow r = 1$ m

Thus, the radius of the base of the cylindrical tank is 1 m.

Example 2: What is the height of a cylinder whose volume is 6.16 m^3 and the diameter of whose base is 28 dm?

Solution: Diameter of the base of the cylinder = 28 dm

∴ Radius (r) of the base $=\left(\frac{28}{2}\right)$ dm

$=14$ dm

$=\left(\frac{14}{10}\right)$ m $\quad\left(\because 1 \text{ dm}=\frac{1}{10}\text{ m}\right)$

$=1.4$ m

Volume of the cylinder = 6.16 m^3

$\Rightarrow \pi r^2 h = 6.16 \text{ m}^3$

$\Rightarrow \frac{22}{7}\times(1.4 \text{ m})^2 \times h = 6.16 \text{ m}^3$

$\Rightarrow h=\left[\frac{6.16\times 7}{22\times(1.4)^2}\right]\text{m}$

$\Rightarrow h$ = 1 m

Thus, the height of the cylinder is 1 m.

Example 3: The external diameter, thickness and length of a cylindrical water pipe are 22 cm, 1 cm, and 8 m respectively. What amount of material went into making this pipe?

Solution:

External diameter of the hollow cylindrical pipe = 22 cm

∴ External radius, R = $\left(\frac{22}{2}\right)\text{cm}=11\text{ cm}$

Thickness of the pipe = 1 cm

∴ Internal radius, r = (11 − 1) cm = 10 cm

Length (h) of the pipe = 8 m = (8 × 100) cm = 800 cm ($\because$ 1 m = 100 cm)

$\therefore$ Volume of the material used to make the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(11^2 - 10^2\right) \times 800\right] \text{cm}^3$$
$$= \left[\frac{22}{7} \times 21 \times 800\right] \text{cm}^3$$
$$= 52800 \text{ cm}^3$$

Thus, 52800 cm^3 of material was used to make the water pipe.

Medium

Example 1: The diameter and height of a solid metallic cylinder are 21 cm and 25 cm respectively. If the mass of the metal is 8 g per cm^3, then find the mass of the cylinder.

Solution: Diameter of the cylinder = 21 cm

$\therefore$ Radius (r) of cylinder = $\left(\frac{21}{2}\right)$cm

Height (h) of the cylinder = 25 cm

To find the mass of the metallic cylinder, we have to first find the volume of the cylinder.

Volume of the cylinder $= \pi r^2 h$

$$= \left[\frac{22}{7} \times \left(\frac{21}{2}\right) \times \left(\frac{21}{2}\right) \times 25\right] \text{cm}^3$$
$$= 8662.5 \text{ cm}^3$$

Mass of 1 cm^3 of the metal = 8 g

$\therefore$ Mass of 8662.5 cm^3 of the metal= (8662.5 × 8) g

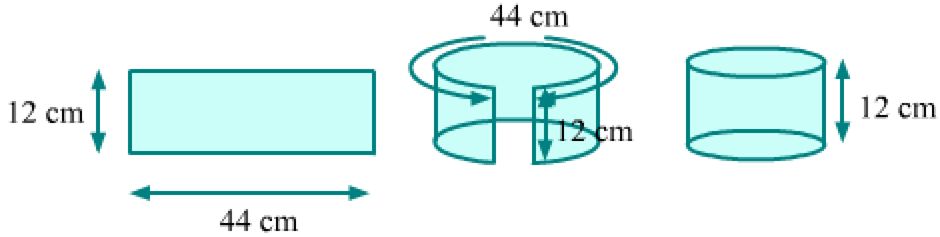
$$= 69300 \text{ g}$$
$$= \left(\frac{69300}{1000}\right) \text{kg} \qquad \left(\because 1 \text{ g} = \frac{1}{1000} \text{kg}\right)$$
$$= 69.3 \text{ kg}$$

Thus, the mass of the cylinder is 69.3 kg.

Example 2: A rectangular sheet of paper is folded to form a cylinder of height 12 cm. If the length and breadth of the sheet are 44 cm and 12 cm respectively, then find the volume of the cylinder.

Solution: Height (*h*) of the cylinder = 12 cm

Let *r* be the radius of the cylinder. We can find this value from the circumference of the base of the cylinder. As shown in the figure, this circumference is nothing but the length of the sheet.



So, circumference of the base of the cylinder = 44 cm

$$\Rightarrow 2\pi r = 44$$
$$\Rightarrow r = \frac{44}{2\pi}$$
$$\Rightarrow r = \frac{44\times 7}{2\times 22}$$

$$\Rightarrow r = 7cm$$

Now, volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7}\times 7\times 7\times 12 \text{ cm}^3$$
$$= 1848 \text{ cm}^3$$

Hard

Example 1: The inner and outer diameters of a cylindrical iron pipe are 54 cm and 58 cm respectively and its length is 5 m. What is the mass of the pipe if 1 cm^3 of iron has a mass of 8 g?

Solution: Inner diameter of the hollow cylindrical iron pipe = 54 cm

∴ Inner radius, $r = \left(\frac{54}{2}\right)\text{cm} = 27\text{ cm}$

Outer diameter of the pipe = 58 cm

∴ Outer radius, $R = \left(\frac{58}{2}\right)\text{cm} = 29\text{ cm}$

Length (h) of the pipe = 5 m = (5 × 100) cm = 500 cm

∴ Volume of the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(29^2 - 27^2\right) \times 500\right]\text{cm}^3$$
$$= \left[\frac{22}{7} \times 112 \times 500\right]\text{cm}^3$$
$$= 176000\text{ cm}^3$$

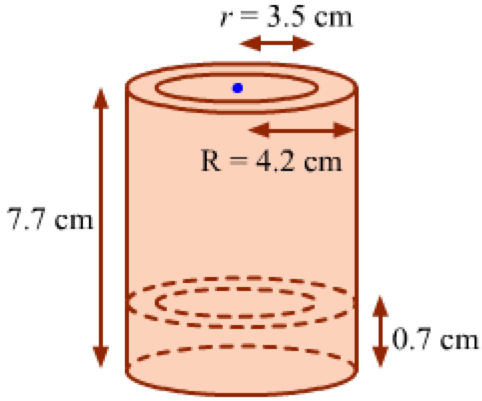
Mass of 1 cm^3 of iron = 8 g

∴ Mass of 176000 cm^3 of iron = (8 × 176000) g

$$= \left(\frac{8 \times 176000}{1000}\right)\text{kg} \qquad \left(\because 1\text{ g} = \frac{1}{1000}\text{kg}\right)$$
$$= 1408\text{ kg}$$

Thus, the mass of the hollow cylindrical iron pipe is 1408 kg.

Example 2: The internal and external radii of a cylindrical juice can (as shown in the figure) are 3.5 cm and 4.2 cm respectively. The total height of the can is 7.7 cm. The thickness of the base (i.e., a solid cylinder) is 0.7 cm. If the mass of the material used in the can is 3 g per cm^3, then find the mass of the can.



Solution: To find the mass of the juice can, we need to first find its volume.

The juice can shown in the figure contains two cylinders. One is a solid cylinder (i.e., the base of the can) and the other is a hollow cylinder (i.e., the cylindrical part that stands on the base).

External radius (R) of the hollow cylinder = 4.2 cm

Internal radius (r) of the hollow cylinder = 3.5 cm

Thickness (h) of the base = 0.7 cm (i.e., the height of the solid cylinder)

Total height (H) of the juice can = 7.7 cm

∴ Height (h') of the hollow cylinder = (7.7 − 0.7) cm = 7 cm

Volume of the juice can = Volume of the solid base + Volume of the hollow cylinder on the base

$$= \pi R^2 h + \pi\left(R^2 - r^2\right)h'$$

$$= \pi\left[R^2 h + \left(R^2 - r^2\right)h'\right]$$

$$= \frac{22}{7}\left[(4.2)^2 \times 0.7 + \left\{(4.2)^2 - (3.5)^2\right\} \times 7\right]\text{cm}^3$$

$$= \frac{22}{7}(12.348 + 5.39 \times 7)\text{cm}^3$$

$$= \frac{22}{7}(12.348 + 37.73)\text{cm}^3$$

$$= \left(\frac{22}{7} \times 50.078\right)\text{cm}^3$$

$$= 157.388 \text{ cm}^3$$

Mass of the material per cm^3 = 3 g

∴ Mass of the material used in the container = (3 × 157.388) g

= 472.164 g

Thus, the mass of the juice can is 472.164 g.

Example 3: A well 3.5 m in diameter and 20 m deep is dug in a rectangular field of dimensions 20 m × 14 m. The earth taken out is spread evenly across the field. Find the level of earth raised in the field.

Solution:

Length (l) of the field = 20 m

Breadth (b) of the field = 14 m

Diameter (d) of the well = 3.5 m

∴ Radius (r) of the well= $\frac{3.5}{2}$ m

Depth (h) of the well = 20 m

Volume of the dug out earth = $\pi r^2 h$

Now, the area of the field on which the dug out earth is spread is given by the difference between the area of the entire field and the area of the field covered by the cross-section of the well.

$\Rightarrow l \times b - \pi r^2$

Let H be the level of earth raised in the field.

Volume of earth spread in the field = Volume of the dug out earth

$$\Rightarrow \left(l \times b - \pi r^2\right) H = \pi r^2 h$$

$$\Rightarrow \left(20 \times 14 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) H = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 20$$

$$\Rightarrow \left(280 - \frac{269.5}{28}\right) H = \frac{5390}{28}$$

$$\Rightarrow \left(280 - \frac{77}{8}\right) H = \frac{385}{2}$$

$$\Rightarrow \frac{2163}{8} H = \frac{385}{2}$$

$$\Rightarrow H = \frac{385}{2} \times \frac{8}{2163}$$

$$\Rightarrow H = 0.71197 \text{ m} \approx 0.712 \text{ m}$$

Therefore, the level of earth in the field is raised by about 0.712 m.

Volume of Cone

Ice creams are loved by one and all. Take a look at the one shown.

Clearly, what is shown above is an ice cream cone, i.e., ice cream inside a crisp conical wafer. The amount of ice cream present in the cone is equal to the volume of the cone. In other words, the number of cubic units of ice cream that will exactly fill the cone is the volume of the cone.

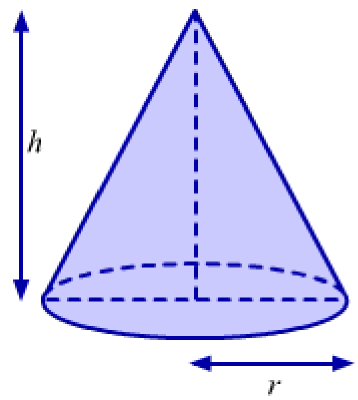
In this lesson, we will learn the formula for the volume of a right circular cone and solve examples using the same.

Did You Know?

A waffle maker named Ernest Hamwi is credited by a few to be the inventor of the ice cream cone. He is said to have come up with the idea in 1904 to help an ice cream vendor who had run out of dishes to serve ice cream.

Formula for the Volume of a Right Circular Cone

Consider a cone of radius *r* and height *h*.



The formula for the volume of this right circular cone is given as follows:

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Using the above formula, we can find the cubic units of ice cream that exactly fill a cone.

Let us say the radius and height of an ice cream cone are 3.5 cm and 9 cm respectively. Then,

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 9\text{ cm}^3 = 115.5\text{ cm}^3$

Thus, the amount of ice-cream that exactly fills the cone is 115.5 cm^3.

Did You Know?

For a cone and a cylinder with the same base radius and height, the volume of the cone is one-third that of the cylinder.

Solved Examples

Easy

Example 1: The height and slant height of a conical funnel are 21 cm and 29 cm respectively. How many litres of water can the funnel hold?

Solution:

The amount of water that the funnel can hold is equal to the volume of the funnel.

Height (h) of the funnel = 21 cm

Slant height (l) of the funnel= 29 cm

Let the radius of the circular base of the funnel be r.

Now, $l^2 = r^2 + h^2$

$\Rightarrow (29 \text{ cm})^2 = r^2 + (21 \text{ cm})^2$

$\Rightarrow 841 \text{ cm}^2 = r^2 + 441 \text{ cm}^2$

$\Rightarrow r^2 = (841 - 441) \text{ cm}^2$

$\Rightarrow r^2 = 400 \text{ cm}^2$

$\Rightarrow r = \sqrt{400} \text{ cm} = 20 \text{ cm}$

Volume of the funnel $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 21\right) \text{cm}^3$$

$$= 8800 \text{ cm}^3$$

$$= \frac{8800}{1000} \text{L} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{L}\right)$$

$$= 8.8 \text{ L}$$

Thus, the funnel can hold 8.8 L of water.

Example 2: If A, B and C are respectively the height, volume and curved surface area of a cone, then prove that $3B(\pi A^3 + 3B) = C^2A^2$.

Solution: It is given that A, B and C are respectively the height, volume and curved surface area of the cone.

Let r and l be the radius and slant height of the cone.

Now,

$$B = \frac{1}{3}\pi r^2 A \qquad ...(1)$$

$$C = \pi r l \qquad ...(2)$$

$$l = \sqrt{r^2 + A^2} \qquad ...(3)$$

We have to prove $3B(\pi A^3 + 3B) = C^2A^2$. Let us take the LHS of this equation.

$$3B\left(\pi A^3 + 3B\right)$$
$$= 3\pi BA^3 + 9B^2$$
$$= 3\pi\left(\frac{1}{3}\pi r^2 A\right) \times A^3 + 9\left(\frac{1}{3}\pi r^2 A\right)^2 \qquad \text{(Using equation 1)}$$
$$= \pi^2 r^2 A^4 + \pi^2 r^4 A^2$$
$$= \pi^2 r^2 A^2\left(A^2 + r^2\right)$$
$$= \pi^2 r^2 A^2 \times l^2 \qquad \text{(Using equation 3)}$$
$$= \pi^2 r^2 l^2 \times A^2$$
$$= C^2 A^2 \qquad \text{(Using equation 2)}$$
$$= \text{RHS}$$

Medium

Example 1: The radius and slant height of a cone are in the ratio 3 : 5. If the volume of the cone is 12936 m^3, then find the radius, height and slant height of the cone.

Solution: Let the radius (r) and slant height (l) of the cone be $3x$ and $5x$ respectively.

Let the height of the cone be h.

We know that $l^2 = r^2 + h^2$

$\Rightarrow (5x)^2 = (3x)^2 + h^2$

$\Rightarrow h^2 = 25x^2 - 9x^2$

$\Rightarrow h^2 = 16x^2$

$\Rightarrow h = \sqrt{16x^2} = 4x$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

It is given that the volume of the cone is 12936 m^3.

So, $\frac{1}{3}\pi r^2 h = 12936 \text{ m}^3$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (3x)^2 \times (4x) = 12936 \text{ m}^3$$
$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 \times x^3 = 12936 \text{ m}^3$$
$$\Rightarrow x^3 = \frac{12936 \times 3 \times 7}{22 \times 9 \times 4} \text{m}^3$$
$$\Rightarrow x^3 = 343 \text{ m}^3$$
$$\Rightarrow x^3 = (7 \text{ m})^3$$

$$\Rightarrow x = 7m$$

Now, $r = 3x = (3 \times 7)$ m = 21 m

$h = 4x = (4 \times 7)$ m = 28 m

$l = 5x = (5 \times 7)$ m = 35 m

Thus, the radius, height and slant height of the cone are 21 m, 28 m and 35 m respectively.

Example 2: If the radii and heights of two cones are in the ratios 2 : 3 and 5 : 4 respectively, then find the ratio of the volumes of the cones.

Solution: Let r_1 and h_1 be the radius and height of one cone.

Let r_2 and h_2 be the radius and height of the other cone.

It is given that the radii of the cones are in the ratio 2 : 3.

$\therefore \frac{r_1}{r_2} = \frac{2}{3}$

It is given that the heights of the cones are in the ratio 5 : 4.

$\therefore \frac{h_1}{h_2} = \frac{5}{4}$

Ratio of the volumes of the cones $= \frac{\text{Volume of the first cone}}{\text{Volume of the second cone}}$

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$$
$$= \left(\frac{2}{3}\right)^2 \times \left(\frac{5}{4}\right)$$
$$= \frac{4}{9} \times \frac{5}{4}$$
$$= \frac{5}{9}$$

Thus, the volumes of the two cones are in the ratio 5 : 9.

Hard

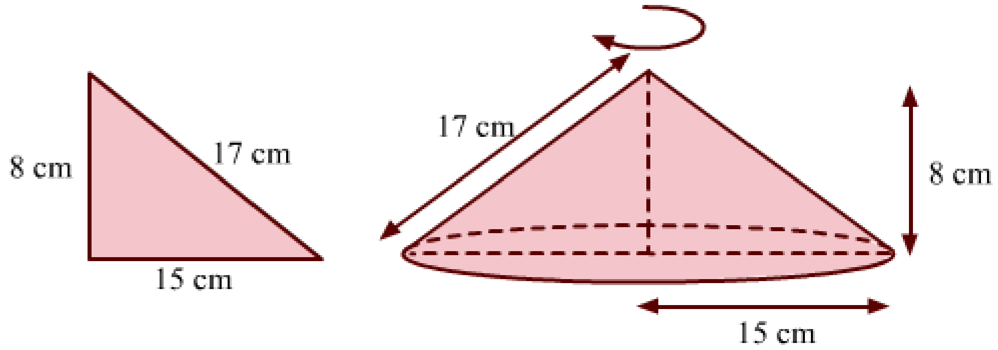
Example 1: Find the volume (in terms of π) of the solid figure obtained when a right triangle with sides 8 cm, 15 cm and 17 cm is revolved about the side

i) 8 cm.

ii) 15 cm.

Solution: i) The sides of the given right triangle are 8 cm, 15 cm and 17 cm.

If this right triangle is revolved about the side 8 cm, then we will obtain a solid figure as is shown.



The solid figure so obtained is a cone.

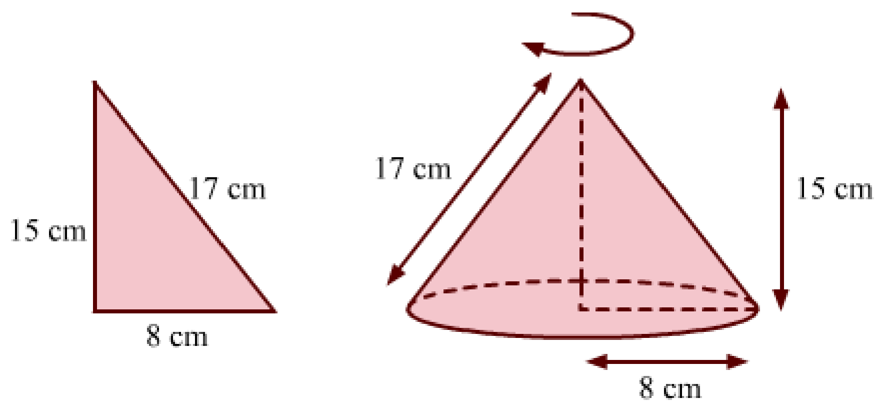
The radius (r)and height (h) of the cone are 15 cm and 8 cm respectively.

∴ Volume of the cone formed $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 15 \times 15 \times 8 \text{ cm}^3$

$= 600\pi \text{ cm}^3$

ii) If the same right triangle is revolved about the side 15 cm, then we will obtain the following solid figure.



Again, the solid figure so obtained is a cone.

The radius (r)and height (h) of the cone are 8 cm and 15 cm respectively.

∴ Volume of the cone formed $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 8 \times 8 \times 15 \text{ cm}^3$$
$$= 320\pi \text{ cm}^3$$

Example 2: The surface area of a sphere of radius 5 cm is five times the curved surface area of a cone of radius 4 cm. Find the

i) height of the cone.

ii) volume of the cone.

Solution:

i) Let r_1 be the radius of the sphere and r_2 be the radius of the cone.

Let h the height and l the slant height of the cone.

It is given that r_1 = 5 cm and r_2 = 4 cm

According to the question, we have:

Surface area of the sphere = 5 × Curved surface area of the cone

$$\Rightarrow 4\pi r_1^2 = 5 \times \pi r_2 l$$
$$\Rightarrow 4 \times (5 \text{ cm})^2 = 5 \times 4 \text{ cm} \times l$$

$$\Rightarrow l = 5cm$$

We know that $l = \sqrt{r_2^2 + h^2}$
$$\Rightarrow (5 \text{ cm})^2 = (4 \text{ cm})^2 + h^2$$
$$\Rightarrow h^2 = 25 \text{ cm}^2 - 16 \text{ cm}^2$$
$$\Rightarrow h^2 = 9 \text{ cm}^2$$

$$\Rightarrow h = \sqrt{9}\, cm = 3cm$$

Thus, the height of the cone is 3 cm.

ii) Volume of the cone $= \frac{1}{3} \pi r_2^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 \text{ cm}^3$$
$$= 50.29 \text{ cm}^3$$

Volumes of Spheres and Hemispheres

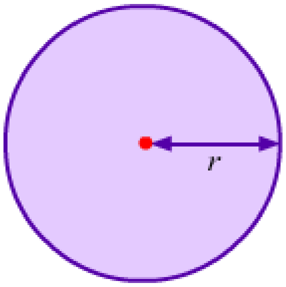
Consider the basketball shown.

Clearly, the basketball, (or, for that matter, any ball) is spherical or shaped like a sphere. Being inflatable, a basketball acquires its shape on being filled with air. The amount of air inside a basketball filled to its capacity helps us ascertain the volume of the ball.

In this lesson, we will learn the formulae for the volumes of spheres and hemispheres, and solve problems using the same.

Formula for the Volume of a Sphere

Consider a solid sphere of radius *r*.



The formula for the volume of this solid sphere is given as follows:

Volume of the solid sphere = $\frac{4}{3}\pi r^3$

Using the above formula, we can calculate the amount of air in a basketball filled to its capacity.

Suppose we have a basketball with radius 18 cm. Then, Volume of the basketball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 18\times 18\times 18\ \text{cm}^3 = 24438.86\ \text{cm}^3$

Thus, the amount of air filled inside the basketball is 24438.86 cm^3.

Solved Examples

Example 1: Find the radius of a sphere if its volume is $179\frac{2}{3}\ \text{cm}^3$.

Solution: Volume of a sphere = $\frac{4}{3}\pi r^3$

It is given that the volume of the given sphere is $179\frac{2}{3}\ \text{cm}^3$.

Let the radius of the given sphere be *r*.

So,

$$\frac{4}{3}\pi r^3 = 179\frac{2}{3}\ \text{cm}^3$$
$$\Rightarrow \frac{4}{3}\times\frac{22}{7}r^3 = \frac{539}{3}\ \text{cm}^3$$
$$\Rightarrow r^3 = \frac{539\times 7\times 3}{4\times 22\times 3}\ \text{cm}^3$$
$$= \left(\frac{7}{2}\right)^3\ \text{cm}^3$$

$$\Rightarrow r = \frac{7}{2}cm = 3.5cm$$

Thus, the radius of the sphere is 3.5 cm.

Medium

Example 1: Find the volume of a sphere if its surface area is 154 cm^2.

Solution: Surface area of a sphere = $4\pi r^2$

It is given that the surface area of the given sphere is 154 cm^2.

Let the radius of the given sphere be *r*.

So,

$$4\pi r^2 = 154$$
$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$$
$$\Rightarrow r^2 = \frac{154 \times 7}{22 \times 4}$$
$$\Rightarrow r^2 = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2} cm$$

Now, volume of a sphere = $\frac{4}{3}\pi r^3$

Therefore, volume of the given sphere = $\frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^3$

= 179.67 cm^3

Example 2: The diameter of Earth is about $\mathbf{\frac{20}{19}}$ times that of Venus. What is the ratio of their volumes?

Solution: Volume of a sphere = $\frac{4}{3}\pi r^3$

Let the diameter of Venus be *x*.

∴ Diameter of Earth = $\frac{20}{19}x$

Now, radius (r_1) of Venus = $\frac{x}{2}$

And, radius (r_2) of Earth = $\frac{10}{19}x$

Ratio of the volumes of the two planets $= \dfrac{\text{Volume of Earth}}{\text{Volume of Venus}}$

$$= \dfrac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3}$$

$$= \left(\dfrac{r_2}{r_1}\right)^3$$

$$= \left(\dfrac{\frac{10}{19}x}{\frac{x}{2}}\right)^3$$

$$= \left(\dfrac{20}{19}\right)^3$$

$$= \dfrac{8000}{6859}$$

Thus, the ratio of the volumes of Earth and Venus is 8000 : 6859.

Hard

Example 1: Find the number of spherical lead shots each 2.1 cm in diameter which can be obtained from a rectangular solid of lead with dimensions 66 cm × 42 cm × 21 cm.

Solution: Let x number of spherical lead shots be obtained from the given solid of lead.

Volume of a cuboid = $l \times b \times h$

∴ Volume of lead in the given rectangular solid = (66 × 42 × 21) cm^3

Diameter of a lead shot = 2.1 cm

∴ Radius of a lead shot = $\dfrac{2.1}{2}\text{ cm} = 1.05\text{ cm}$

Volume of a sphere = $\dfrac{4}{3}\pi r^3$

∴Volume of a lead shot = $\frac{4}{3}\times\frac{22}{7}\times(1.05)^3\ \text{cm}^3$

⇒ Volume of x number of lead shots = $\frac{4}{3}\times\frac{22}{7}\times x\times(1.05)^3\ \text{cm}^3$

According to the question, we have:

Volume of x number of lead shots = Volume of lead in the rectangular solid

So,

$$\frac{4}{3}\times\frac{22}{7}\times x\times(1.05)^3 = 66\times42\times21$$

$$\Rightarrow x=\frac{66\times42\times21\times3\times7}{4\times22\times(1.05)^3}$$

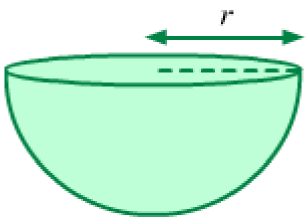
$$\Rightarrow x = 12000$$

Thus, 12000 spherical lead shots can be formed from the given solid of lead.

Formula for the Volume of a Hemisphere

On cutting a solid spherical object into two equal parts, we obtain two solid hemispheres. The radius of each hemisphere so obtained is the same as that of the sphere.

Consider a hemisphere of radius *r*.



Since hemispheres are obtained by cutting a sphere in half, the volume of each resultant hemisphere is equal to half of that of the sphere.

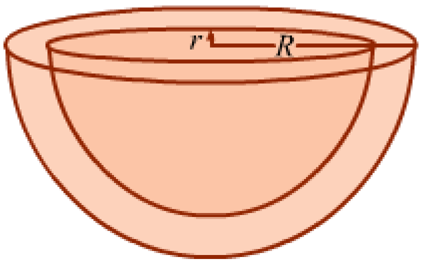
The formula for the volume of this solid hemisphere is arrived at as follows:

Volume of the solid hemisphere $=\frac{1}{2}\times\frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

Formula for the Volume of a Hollow Hemisphere

Let *R* and *r* be the outer and inner radii of the hollow hemisphere.



Volume of a hollow hemisphere = Volume of outer hemisphere − Volume of inner hemisphere

$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \left(R^3 - r^3\right)$

Solved Examples

Easy

Example 1: How many litres of milk can a hemispherical bowl of diameter 21 cm hold?

Solution: Diameter of the hemispherical bowl = 21 cm

∴ Radius (*r*) of the hemispherical bowl= $\frac{21}{2}$ cm

Volume of the hemispherical bowl = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^3$$
$$= 2425.5 \text{ cm}^3$$
$$= \frac{2425.5}{1000} \text{ L} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L} \right)$$
$$= 2.4255 \text{ L}$$
$$\approx 2.43 \text{ L}$$

Thus, the hemispherical bowl can hold approximately 2.43 L of milk.

Medium

Example 1: A hemispherical bowl is made of one-centimetre-thick steel. The inside radius of the bowl is 6 cm. Find the volume of steel used in making the bowl.

Solution:

Inner radius (r) of the hemispherical bowl = 6 cm

Outer radius (R) of the bowl = (6 + 1) cm = 7 cm (∵ Steel used has thickness of 1 cm)

Volume of the inner hemisphere = $\frac{2}{3}\pi r^3$

Volume of the outer hemisphere = $\frac{2}{3}\pi R^3$

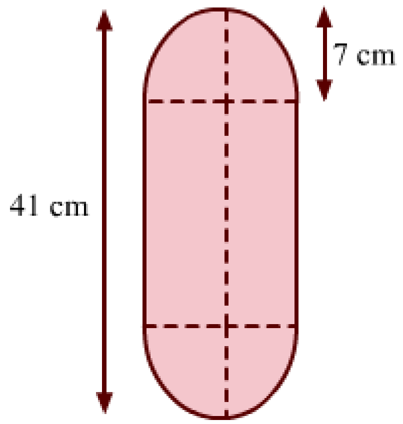
∴ Volume of steel used $= \frac{2}{3}\pi \left(R^3 - r^3 \right)$

$$= \frac{2}{3} \times \frac{22}{7} \times \left[(7)^3 - (6)^3 \right] \text{ cm}^3$$
$$= \frac{2}{3} \times \frac{22}{7} \times (343 - 216) \text{ cm}^3$$
$$= \frac{2}{3} \times \frac{22}{7} \times 127 \text{ cm}^3$$
$$= 266.095 \text{ cm}^3$$

Thus, the volume of steel used in making the bowl is 266.095 cm^3.

Hard

Example 1: A solid is in the form of a cylinder with hemispherical ends as is shown in the figure. Find the volume of the solid.



Solution:

It is given that:

Radius (r) of the cylinderical part of the solid = 7 cm

Height (h) of the same = [41 − (2 × 7)] cm = (41 − 14) cm = 27 cm

Also, radius of each hemispherical part is the same as that of the cylinderical part.

∴ Volume of the solid = Volume of the cylinderical part + Volumes of the hemispherical parts

$$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$$
$$= \pi r^2\left(h + \frac{4r}{3}\right)$$
$$= \frac{22}{7}\times 7\times 7\times\left(27 + \frac{4\times 7}{3}\right)\text{cm}^3$$
$$= \frac{22}{7}\times 7\times 7\times\frac{109}{3}\text{cm}^3$$
$$= 5595.33\ \text{cm}^3$$